**SNSB**
**Summer Term 2013**
**Ergodic Theory and Additive Combinatorics**
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# Seminar 3

**(S3.1)**

(i) $(X, 1_X)$ is minimal if and only if $|X| = 1$.

(ii) If $(X, T)$ is minimal, then $T$ is surjective.

(iii) A factor of a minimal TDS is also minimal.

(iv) If a product TDS is minimal, then so are each of its components.

(v) If $(X_1, T_{X_1})$, $(X_2, T_{X_2})$ are two minimal subsystems of a TDS $(X, T)$, then either $X_1 \cap X_2 = \emptyset$ or $X_1 = X_2$.

(vi) A disjoint union of two TDSs is never a minimal TDS.

**(S3.2)** Let $(X, T)$ be a TDS and assume that $X$ is metrizable. For any $x \in X$, the following are equivalent:

(i) $x$ is recurrent.

(ii) $\lim_{k\to\infty} T^{n_k}x = x$ for some sequence $(n_k)$ in $\mathbb{Z}_+$.

(iii) $\lim_{k\to\infty} T^{n_k}x = x$ for some sequence $(n_k)$ in $\mathbb{Z}_+$ such that $\lim_{k\to\infty} n_k = \infty$.

**(S3.3)**

(i) If $\varphi : (X, T) \to (Y, S)$ is a homomorphism of TDSs and $x \in X$ is recurrent (almost periodic) in $(X, T)$, then $\varphi(x)$ is recurrent (almost periodic) in $(Y, S)$.

(ii) If $(A, T_A)$ is a subsystem of $(X, T)$ and $x \in A$, then $x$ is recurrent (almost periodic) in $(X, T)$ if and only if $x$ is recurrent (almost periodic) in $(A, T_A)$.

**(S3.4)** Let $(X, T)$ be a TDS. The following are equivalent:

(i) $(X, T)$ is minimal.

(ii) every point of $X$ is forward transitive and almost periodic.

(iii) there exists a forward transitive point $x_0 \in X$ which is also almost periodic.

**(S3.5)** Let $(X, T)$ be a TDS and $x \in X$. The following are equivalent:

(i) $x$ is almost periodic.

(ii) For any open neighborhood $U$ of $x$, there exists $N \geq 1$ such that

$$\mathcal{O}_+(x) \subseteq \bigcup_{k=0}^{N} T^{-k}(U).$$

(iii) $(\overline{\mathcal{O}}_+(x), T_{\overline{\mathcal{O}}_+(x)})$ is a minimal subsystem.